6.3

Binomial and Geometric Random Variables

6.3A

Binomial Settings and Binomial Random Variables, Binomial Probabilities

A binomial setting arises when we perform several independent trials of the same chance process and record the number of times that a particular outcome occurs. The four conditions for a binomial setting are:

1. **B**inary? The possible outcomes of each trial can be classified as "success" or "failure".
2. **I**ndependent? Trials must be independent; knowing the result of one trial must not have any effect on the result of any other trial.
3. **N**umber? The number of trials must be fixed in advance.
4. **S**uccess? On each trial, the probability of success must be the same.

Binomial random variable and binomial distribution

The count the number of successes (X) in a binomial setting is a **binomial random variable.** The probability distribution of X is a **binomial distribution** with parameters **n** and **p**, where n is the **number of trials** of the chance process and p is the **probability of a success on any one trial**. The possible values of X are the whole numbers from 0 to n.

What are some examples of probability distributions that are binomial?

Example

Determine whether the random variables below have a binomial distribution. Justify your answer.

a. Roll a fair die 10 times and let X = the number of sixes

① Binary? ✓ S = 6, F = not 6

② Independent? ✓

③ Number? ✓ n = 10

④ Success? ✓ P(S) = 1/6 every time

Yes!

b. Shoot a basketball 20 times from various distances on the court.
Let Y = number of shots made

① Binary? ✓ S = made , F = miss

② Indep.? ✓

③ # ? ✓ n = 20

④ Success? NO.

No!

c. Observe the next 100 cars that go by and let C = color

① Binary? No. Multiple colors!

No!

Example

In many games involving dice, rolling a 6 is desirable. The probability of rolling a six when rolling a fair die is 1/6. If X = the number of sixes in 4 rolls of a fair die, then X is binomial with $n = 4$ and $p = 1/6$.

trials

P(S) on any trials

a) What is P(X = 0)?

$P(X=0) = \left(\frac{5}{6}\right)^4$

get 0 6s

=

F F F F

b) What is P(X = 1)?

S F F F

F S F F

F F S F

F F F S

$P(X=1) = 4\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^3$

$= \boxed{0.386}$

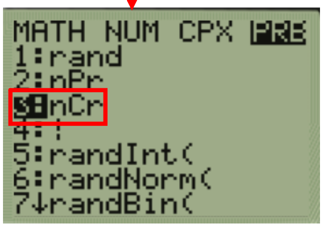
Binomial coefficient

The number of ways of arranging k successes among n observations is given by:

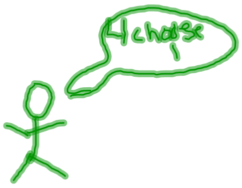
$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

**You are not expected to calculate binomial coefficients by hand on the AP Exam!

*n choose k



Ex. $\binom{4}{1} = \frac{4!}{1!(3)!}$



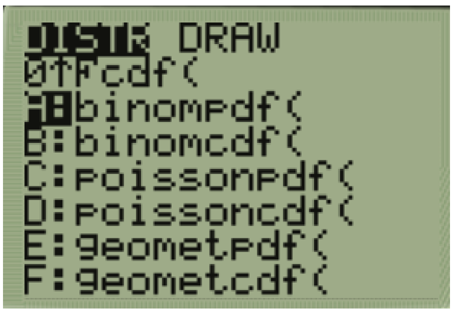
$= \frac{4\cdot 3\cdot 2\cdot 1}{1\cdot 3\cdot 2\cdot 1} = 4$

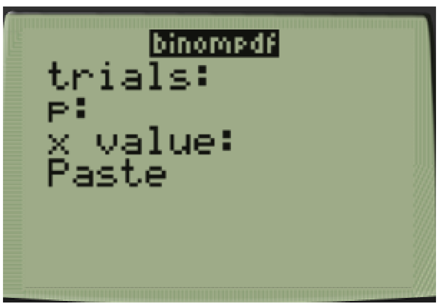
Binomial Probability

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

*This can be done on your calculator. However, you must show that the situation is binomial and write the formula with correct values plugged in.

$P(X=1) = \binom{4}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^3$





Mean and Standard Deviation of a Binomial Random Variable

$\mu_X = np$

$\sigma_X = \sqrt{np(1-p)}$

*Be able to find and interpret in context!!

10% Condition:

When taking an SRS of size n from a population of size N, we can use a binomial distribution to model the count of successes in the sample as long as $n \le \frac{1}{10}N$

*If you are taking a small sample from a large population, you can assume independence!!

Binomial Distribution

Calculator

Your calculator has a built in function that can determine probabilities for binomial distributions:

- **binompdf(n, p, x)**: used when finding the probability of **exactly x successes**.
- **binomcdf(n, p, x)**: used when finding the probability of at **most x successes**. It can also be used to find **at least x successes by utilizing the rules for complements.**

Example

When rolling two dice, the probability of rolling doubles is 1/6. Suppose that a player rolls the dice 4 times, hoping to roll doubles.

a) Find the probability that the player gets doubles twice in 4 attempts.

Binomial:

① Binary ✓ S = Double, F = Not Double

② Ind ✓

③ Number ✓ Trials = 4

④ Success ✓ P(S) = 1/6

i) Formula

$$P(X=k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$$

$n = 4 \quad p = \frac{1}{6} \quad k = 2$

$$P(X=2) = \binom{4}{2} \cdot \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$$

(4 choose 2)

$= .1157$

Calculator:

$X \sim \text{Binomial}(4, \frac{1}{6})$ (n = 4, p = 1/6)

$P(X = 2) = 0.1157$

binompdf(4, 1/6, 2)

b) Should the player be surprised if he gets doubles more than twice
in four attempts?

$P(X=3) + P(X=4)$

$n=4$ ⠀ " "
$p=1/6$ ⠀ " "
$k=3$ ⠀ $k=4$

$P(X>2) = 0.0154 + 0.00077$

$= 0.02$ (5%)

Yes

binomcdf(n, p, k)

↳ $P(X \le k)$

$P(X>2) = 1 - P(X \le 2)$ *binomcdf

$= 1 - 0.98$

$= 0.02$

Example

In Roulette, 18 of the 38 spaces on the wheel are black. Suppose you observe the next 10 spins of a roulette wheel.

(a) What is the probability that exactly 4 of the spins land on black?

(b) What is the probability that at least 8 of the spins land on black?

6.3B

Mean and Standard Deviation of a Binomial Distribution, Binomial Distributions in Statistical Sampling

Example

The makers of a diet cola claim that its taste is indistinguishable from the full-calor version of the same cola. To investigate, an AP Statistics student prepared small samples of each type of soda in identical cups. Then they had volunteers taste each cola in a random order to try to identify which was the diet cola and which was the regular cola. Overall, 23 of the 30 subjects made the correct identification. If we assume that the volunteers really couldn't tell the difference, then each one was guessing with a ½ chance of being correct. Let X = the number of volunteers who correctly identify the colas.

a) Explain why X is a binomial random variable

b) Find the mean and the standard deviation of X. Interpret each value in context.

c) Of the 30 volunteers, 23 made correct identifications. Does this give convincing evidence that the volunteers can taste the difference between the diet and regular colas?

Example

Almost everyone has one – a drawer that holds miscellaneous batteries of all sizes. Suppose that your drawer contains 8 AAA batteries but only 6 of them are good. You need to choose 4 for your graphing calculator. If you randomly select 4 batteries, what is the probability that all 4 of the batteries you choose will work?

Example

A cereal company puts 1 of 5 different NASCAR cards into each box of cereal. Each card features a different driver: Jeff Gordon, Dale Earnhardt, Jr., Tony Stewart, Danica Patrick, or Jimmie Johnson. Suppose that the company printed 20,000 of each card, so there were 100,000 total boxes of cereal with a card inside. If a person bought 6 boxes at random, what is the probability of getting no Danica Patrick cards?

6.3C

Geometric Random Variables

Geometric Setting - A geometric setting arises when we perform independent trial of the same chance process and record the number of trials until a particular outcome occurs. The four conditions are:

1. **B**inary?
2. **I**ndependent?
3. **T**rials? The goal is to count the number of trials until the first success occurs.
4. **S**uccess?

Geometric random variable and geometric distribution

The number of trials Y that it takes to get a success in a geometric setting is a geometric random variable. The probability distribution of Y is a**geometric distribution** with parameter **p**, the probability of a success on any trial. The possible values of Y are 1, 2, 3, …

Geometric Probability

If Y has the geometric distribution with probability p of success on each trial, possible values of Y are 1, 2, 3, …… If k is any one of these values,

$$P(Y = k) = (1 - p)^{k-1} * p$$

Mean (Expected Value) of a Geometric Random Variable

$$E(Y) = \mu_Y = \frac{1}{p}$$

*The formula for geometric probability and mean are NOT on the formula sheet provided on the AP exam. Geometric probabilities can be found using the complement rule and the multiplication rule for independent events.

Example

In the board game Monopoly, one way to get out of jail is to roll doubles.

a) Verify that this is a geometric setting.

b) Find the probability that it takes 3 turns to roll doubles.

c) Find the probability that it takes more than 3 turns to roll doubles, and interpre this value in context.

d) How many rolls would it take, on average, to get out of jail? What does this probability tell you about the shape of the distribution?