

6.1

Discrete and Continuous Random Variables

6.1A

Discrete random Variables, Mean (Expected Value) of a Discrete Random Variable

Random variable - Takes numerical values that describe the outcomes of some chance process

Number of children in a family.

The Friday night attendance at a cinema.

The number of patients in a doctor's surgery.

The number of defective light bulbs in a box of ten.

Probability distribution - Describes the possible values a variable can take and how often it takes those values.

Suppose we flip a coin 3 times and the sample space is as follows:

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

Define the random variable X = the number of heads obtained

Value X	0	1	2	3
Probability $P(X)$				

Example

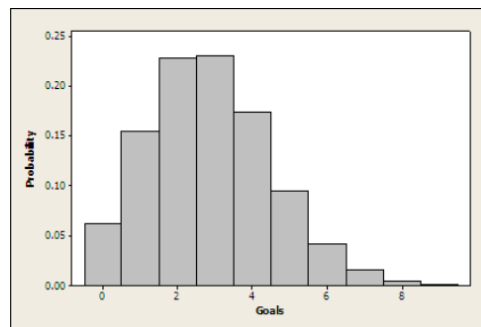
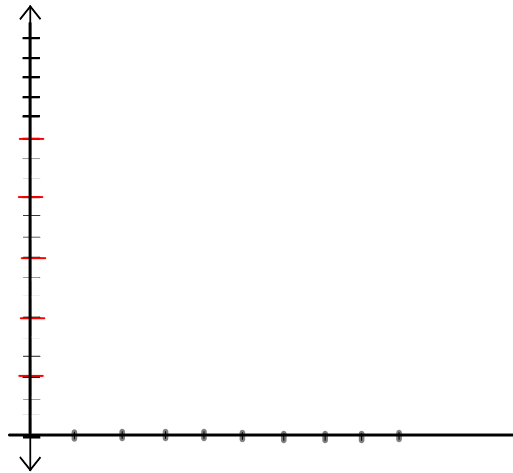
In 2010, there were 1319 games played in the NHL's regular season. Imagine selecting one of these games at random and then randomly selecting one of the two teams that played in the game. Define the random variable X = number of goals scored by a randomly selected team in a randomly selected game. The table below gives the probability distribution of X :

Goals -X	0	1	2	3	4	5	6	7	8	9
Probability - P(X)	0.061	0.154	0.228	0.229	0.173	0.094	0.041	0.015	0.004	0.001

a) Show that the probability distribution for X is legitimate.

All of the probabilities are between 0 and 1 and they add to 1, so this is a legitimate probability distribution.

b) Make a histogram of the probability distribution. Describe what you see.



c) Describe $P(X \geq 6)$ in words and find its value?

$$P(X \geq 6) = 0.041 + 0.015 + 0.004 + 0.001 = 0.061$$

The probability that a randomly selected team scored at least 6 goals is 0.061.

d) What is the probability that a randomly selected team scores more than 6 goals in a game?

$$P(X > 6) = 0.015 + 0.004 + 0.001 = 0.02$$

Mean (Expected Value) of a Discrete random variable -

“What can we expect in the long run?”

- It is an average of the possible outcomes, but a weighted average in which each outcome is weighted by its probability
- Included on the AP Exam formula sheet
- The expected value may NOT be equal to one of the possible values of the variable

$$\mu_x = E(x) = x_1p_1 + x_2p_2 + x_3p_3 + \dots$$

$$\sum x_n p_n$$

Multiply the value of the random variable by its assigned probability and add them together.

6.1B Standard Deviation (and Variance) of a Discrete Random Variable, Continuous Random Variables

Variance and standard deviation of a Discrete random variable -

How much the values of the variable tend to vary, on average, from the expected value. The average distance the outcomes are from the mean.

$$\sigma_x = \sqrt{\sum (x_i - \mu_x)^2 p_i}$$

sigma

σ_x

$$\text{Var}(x) = \sum (x_i - \mu_x)^2 p_i$$

↓ sheet

Example

A wager that players can make in roulette is called a “corner bet.” To make this bet, a player places his chips on the intersection of four numbered squares on the roulette table. If one of these numbers comes up on the wheel and the player bets \$1, the player gets his \$1 back plus \$8 more. Otherwise, the casino keeps the original \$1 bet. If X = the net gain from a single \$1 corner bet, the possible outcomes are $X = -1$ or $X = 8$. Here is the probability distribution of X for 38 bets.

Value:	-1	8
Probability:	34/38	4/38

a) What is the player’s average gain?

Example

Refer to Example #1(NHL) and compute the mean and the standard deviation of the random variable X and interpret these values in context.

The image shows a TI-84 calculator screen. At the top, list editor L1 is labeled 'X', L2 is labeled 'P(X)', and L3 is labeled '1'. Below this, the '1-Var Stats' menu is open, showing options: 1:1-Var Stats, 2:2-Var Stats, 3:Med-Med, 4:LinReg(ax+b), 5:QuadReg, 6:CubicReg, 7:QuartReg. To the right, a smaller window shows 'List:L1', 'FreqList:L2', and 'Calculate'.

$$\underline{E(X)} = \mu_x = 2.9 \quad \text{calc } \bar{x}$$

Expected Value

The avg. # of goals scored in a random game by a random team is 2.9 goals.

$$\sigma_x = 1.6$$

On average, the # of goals scored by a randomly selected team in a random game differs from the mean by 1.6 goals.

Continuous Random Variables -

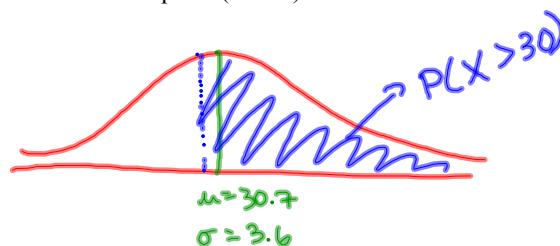
Takes all values in an interval of numbers.

1. The probability distribution of X is described by a density curve.
2. The probability of any event is the area under the density curve and above the values of X that make up the event.
3. Assigns probabilities to intervals of outcomes rather than to individual outcomes.
 - i. All continuous probability models assign probability 0 to every individual outcome. Since each outcome is just one of an infinite number of possible outcomes, the probability is just one of an infinite number of possible outcomes, the probability is $1/\infty$.
4. Usually arise from **MEASURING** something.

Example

The weights of three-year-old females closely follow a Normal distribution with a mean of 30.7 pounds and a standard deviation of 3.6 pounds. Randomly choose one three-year-old female and call her weight X .

- (a) Find and interpret $P(X > 30)$.



$$\text{normcdf}(30, \infty, 30.7, 3.6)$$

$$P(X > 30) = 0.577$$

The probability that a randomly selected 3 yr. old girl has a weight greater than 30 lbs is 0.577.

(b) Find the probability that a randomly selected three-year-old female weighs between 25 and 35 pounds.

$$P(25 < X < 35) = 0.827$$

Example

Joe the barber charges \$32 for a shave and haircut and \$20 for just a haircut. Based on experience, he determines that the probability that a randomly selected customer comes in for a shave and haircut is 0.85, the rest of his customers come in for just a haircut. Let J = what Joe charges a randomly-selected customer.

(a) Give the probability distribution for J .

L1	J	32	20
L2	P(J)	0.85	0.15

(b) Find and interpret the mean of J .

$$E(J) = \mu_J = \$30.20$$

(c) Find and interpret the standard deviation of J .

$$\sigma_J = \$4.28$$

