5.2A

Probability Models

Basic Rules of Probability

Sample Space (S) –

The set of all possible outcomes of a chance process.

When you flip a coin the sample space is {H, T}.

Probability model –

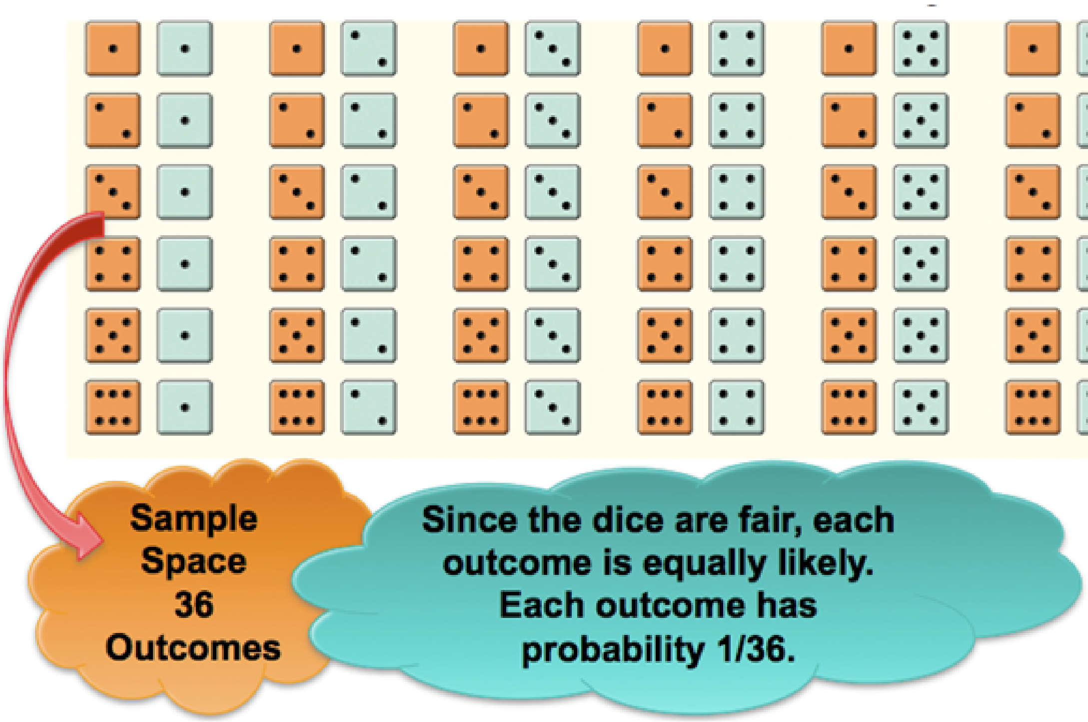
Description of some chance process that consists of two parts: A sample space S and probability for each outcome.

When you flip a coin, S = {H, T} and the P(Heads) = P(Tails) = 1/2

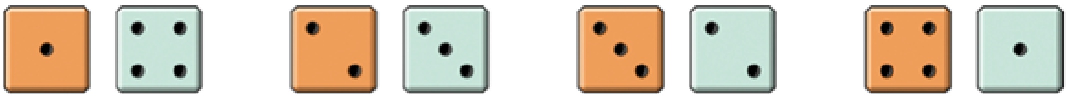
Event –

Any collection of outcomes from some chance process. An event is a subset of the sample space. Events are usually designated by capital letters.

When you flip a coin Event A can be defined as Heads.



In the dice-rolling example, suppose we define event *A* as "sum is 5."



There are 4 outcomes that result in a sum of 5.
Since each outcome has probability 1/36, P(*A*) = 4/36.

Example

Imagine flipping a coin three times. Give a probability model for this chance process.

a) Define the sample space for this model.

S = {HHH, HHT, HTH, HTT, TTT, TTH, THT, THH}

*Tree Diagram

b) What is the probability of getting no heads?

Let A = {TTT}

Since all outcomes are equally likely, P

(A) = 1/8

b) What is the probability of getting two or more heads?

t A = {HHH, HHT, HTH, THH}
Since all outcomes are equally likely, P(A) = 4/8 = 1/2

Basic Rules of Probability:

1. For any event A, $0 \le P(A) \le 1$.
2. If S is the sample space in a probability model, P(S) = 1
3. In the case of equally likely outcomes:

$$P(A) = \frac{number\_of\_outcomes\_corresponding\_to\_A}{total\_number\_of\_outcomes\_in\_sample\_space}$$

4. **Complement Rule:** $P(A^c) = 1 - P(A)$.

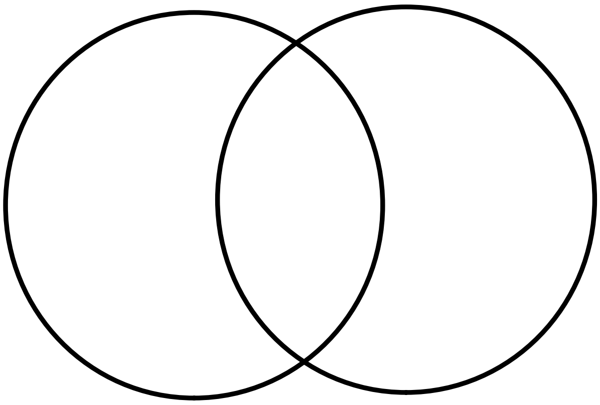
What are complements?

The complement of rolling a 2 on a die is rolling a 1,3,4,5, or 6.

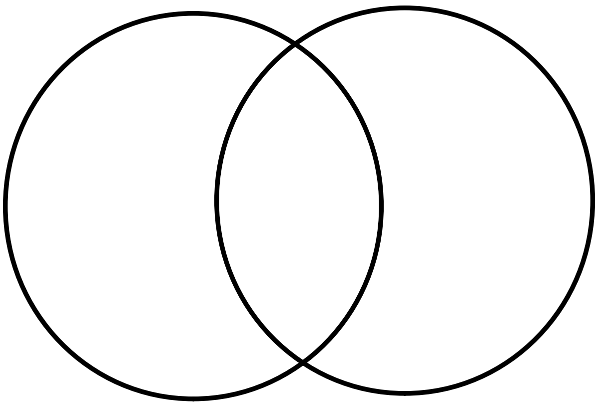
5. **Addition Rule for Mutually Exclusive Events:** P(A or B) = P(A) + P(B)

What does it mean if two events are mutually exclusive?

Raise your hand if you are male. Raise your hand if you are female.



Raise your hand if you are male. Raise your hand if you have brown hair.



6. **General Addition Rule for Two Events:** P(A or B) = P(A) + P(B) – P(A and B)

Example

Randomly select a student who took the 2010 AP Statistics exam and record the student's score. Here is the probability model:

Score	1	2	3	4	5
Probability	0.233	0.183	0.235	0.224	0.125

a) Show that this is a legitimate probability model.

P(S) = 0.233 + 0.183 + 0.235 + 0.224 + 0.125 = 1

All of the probabilities are between 0 and 1 and the sum of the probabilities is 1, so this is a legitimate probability model.

Score	1	2	3	4	5
Probability	0.233	0.183	0.235	0.224	0.125

b) Find the probability that the chosen student scored 3 or better.

By the addition rule, P(3 or better) = 0.235 + 0.224 + 0.125 = 0.584

Score	1	2	3	4	5
Probability	0.233	0.183	0.235	0.224	0.125

c) Find the probability that a student didn't score a 1.

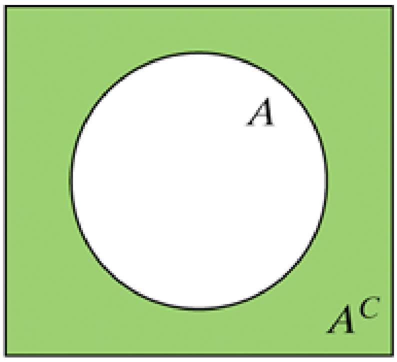
By the complement rule, P(not 1) = 1 - P(1) = 1 - 0.233 = 0.767

5.2B

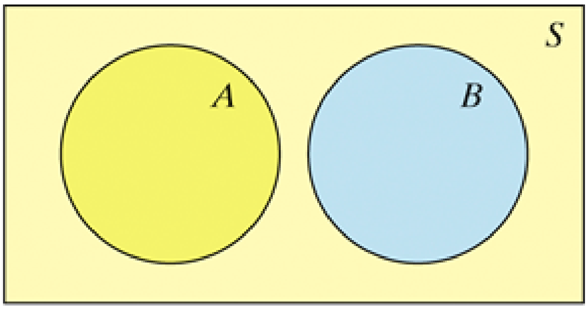
Two-Way Tables and Probability

Venn Diagrams and Probability

The complement A^C contains exactly the outcomes that are not in A.

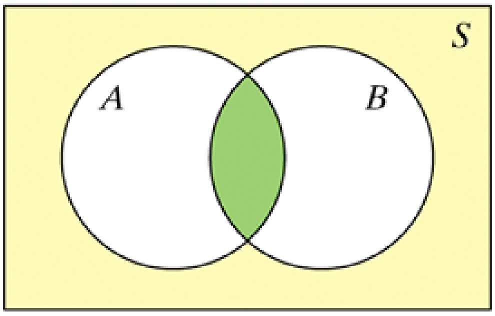


The events *A* and *B* are mutually exclusive (disjoint) because they do not overlap. That is, they have no outcomes in common.

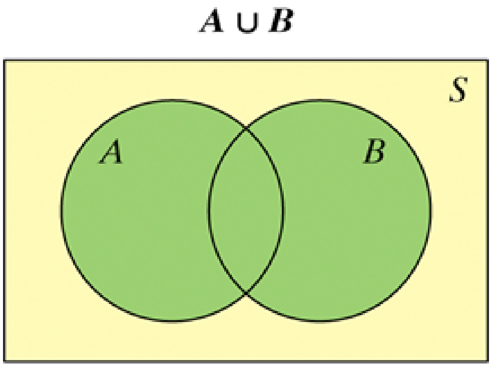


The intersection of events *A* and *B* ($A \cap B$) is the set of all outcomes in both events *A and B*.

$A \cap B$



The union of events A and B ($A \cup B$) is the set of all outcomes in either event *A or B*.



Example

What is the relationship between educational achievement and home ownership? A random sample of 500 people who participated in the 2000 census was chosen. Each member of the sample was identified as a high school graduate (or not) and as a home owner (or not). The two-way table displays the data.

	HS Graduate	Not a HS Graduate	Total
Homeowner	221	119	340
Not a Homeowner	89	71	160
Total	310	190	500

	HS Graduate	Not a HS Graduate	Total
Homeowner	221	119	340
Not a Homeowner	89	71	160
Total	310	190	500

Suppose we choose a member of the sample at random. Find the probability that the member

a) is a high school graduate.

Since 310 of the 500 members of the sample graduated from high school, *P(A)* = 310/500.

	HS Graduate	Not a HS Graduate	Total
Homeowner	221	119	340
Not a Homeowner	89	71	160
Total	310	190	500

b) is a high school graduate and owns a home.

Since 221 of the 500 members of the sample graduated from high school and own a home, *P(A* and *B)* = 221/500.

	HS Graduate	Not a HS Graduate	Total
Homeowner	221	119	340
Not a Homeowner	89	71	160
Total	310	190	500

c) is a high school graduate or owns a home.

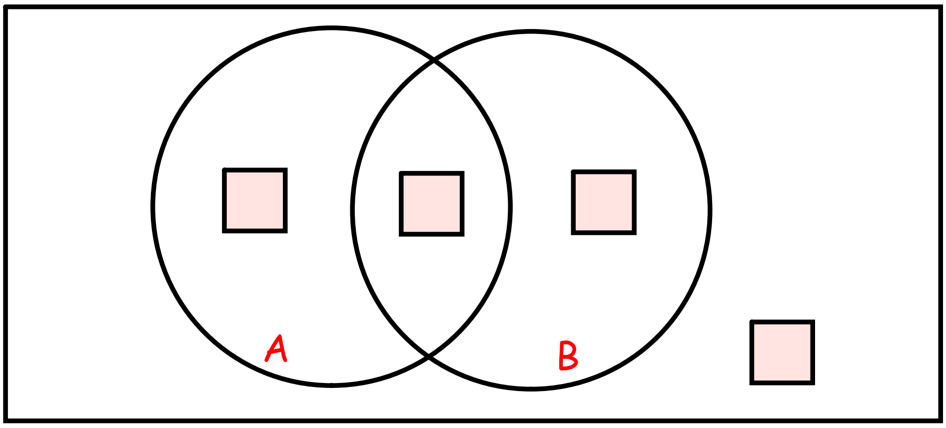
P(HS Graduate or Own Home) = 310/500 + 340/500 - 221/500 = 429/500

*Not Mutually Exclusive!!!

	HS Graduate	Not a HS Graduate	Total
Homeowner	221	119	340
Not a Homeowner	89	71	160
Total	310	190	500

d) Create a Venn diagram illustrating this two-way table

Let A = Homeowner, B = HS Graduate

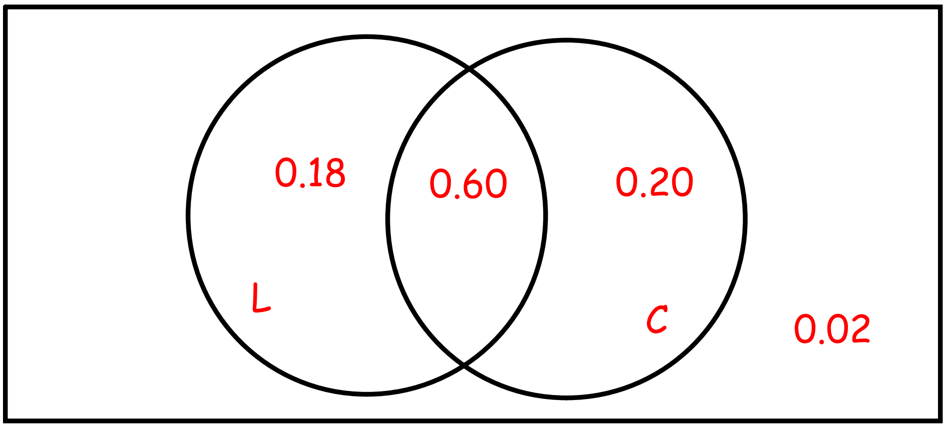


Example

According to the National Center for Health Statistics, in December 2008, 78% of U.S. households had a traditional landline telephone, 80% of households had cell phones, and 60% had both. Suppose we randomly selected a household in December 2008.

a) Make a two-way table that displays the sample space of this chance process

	Cell Phone	No Cell Phone	Total
Landline			
No Landline			
Total			



b) Find the probability that the household has at least one of the two types of phones.

Let A = Cell Phone, B = Landline

P(A or B) = P(A) + P(B) - P(A and B) = 0.80 + 0.78 - 0.60 = 0.98

c) Find the probability that the household has a cell phone only.

P(Cell Phone Only) = 0.20

d) Find the probability that the household has neither type of phone.

P(Neither type of phone) = 0.02